Electrical Technology

Contents

- Resonance in Electric Ckt.
- Series Resonance
- Quality factor of coil at resonance
- Test yourself
- NPTEL Link

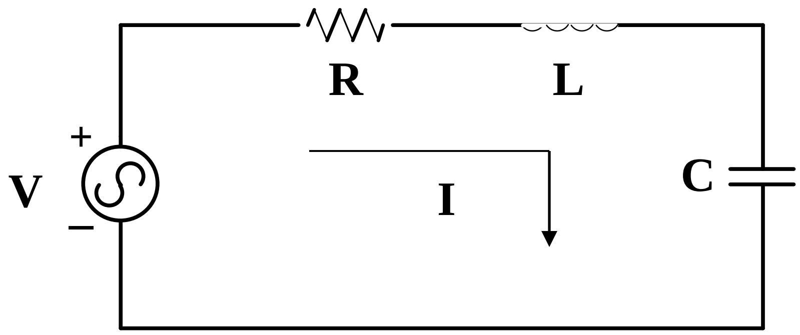
Resonance In Electric Circuits

- Any passive electric circuit will resonate if it has an inductor and capacitor.
- Resonance is characterized by the input voltage and current being in phase. The driving point impedance (or admittance) is completely real when this condition exists.

Series Resonance

Consider the series RLC circuit shown below.

V = $V_M \angle 0$



The input impedance is given by:

$$Z = R + j(wL - \frac{1}{wC})$$

The magnitude of the circuit current is;

$$I = |\overline{I}| = \frac{V_m}{\sqrt{R^2 + (wL - \frac{1}{wC})^2}}$$

Series Resonance

<u>Resonance occurs when,</u>

$$wL = \frac{1}{wC}$$

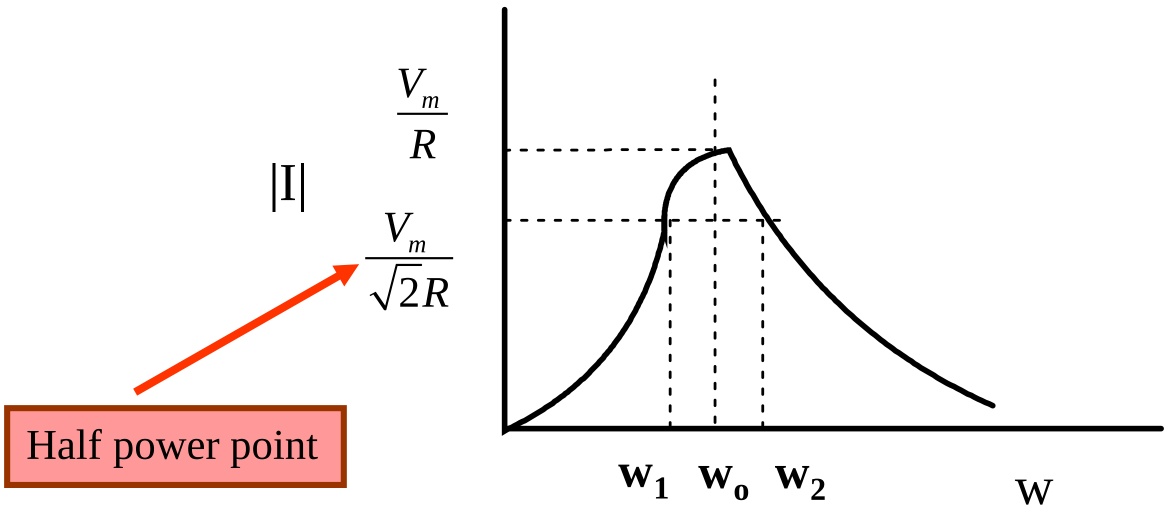
At resonance we designate w as w_o and write;

$$w_o = \frac{1}{\sqrt{LC}}$$

This is *an <u>important equation</u>* to remember. It applies to both series And parallel resonant circuits.

Series Resonance

The magnitude of the current response for the series resonance circuit is as shown below.

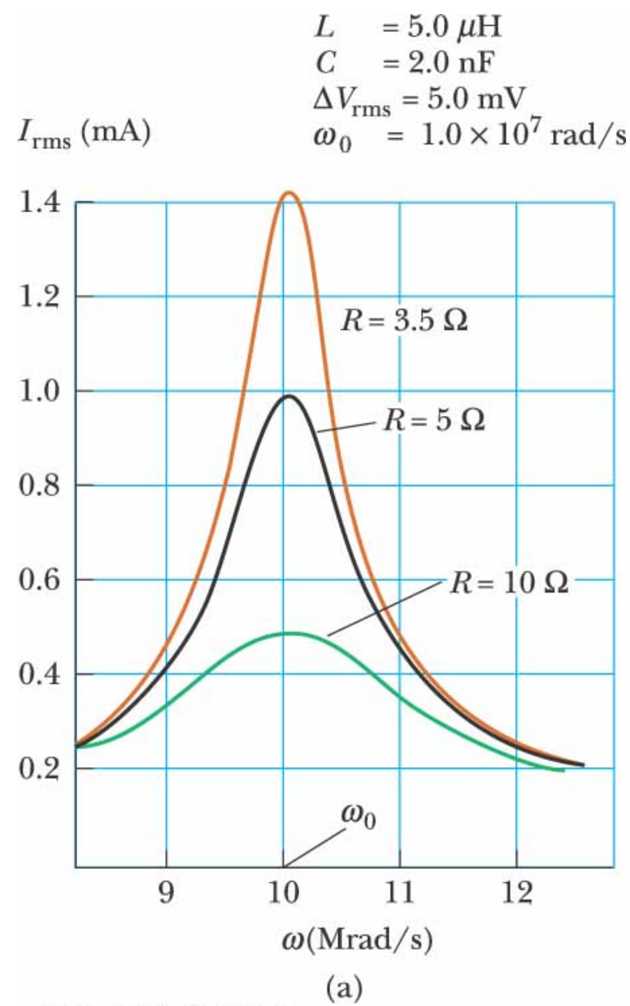


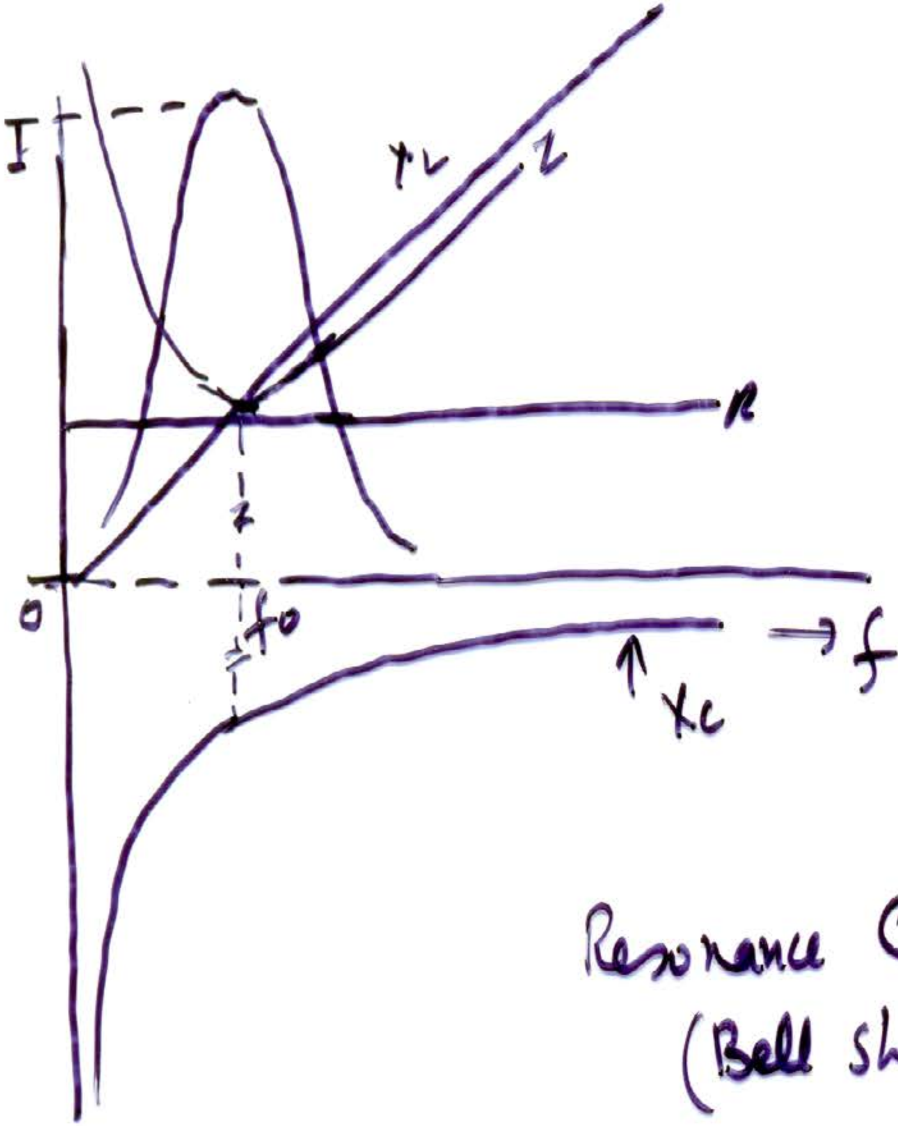
Bandwidth:

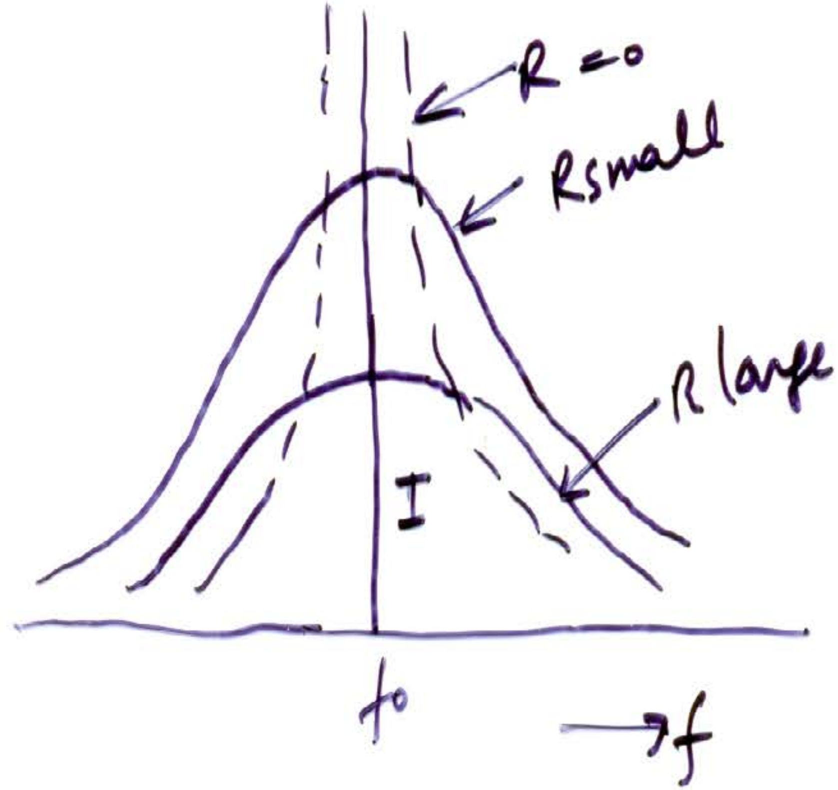
$$BW = w_{BW} = w_2 - w_1$$

Resonance, cont.

- Resonance occurs at the same frequency regardless of the value of *R*
- As *R* decreases, the curve becomes narrower and taller
- Theoretically, if *R* = 0 the current would be infinite at resonance
  - Real circuits always have some resistance



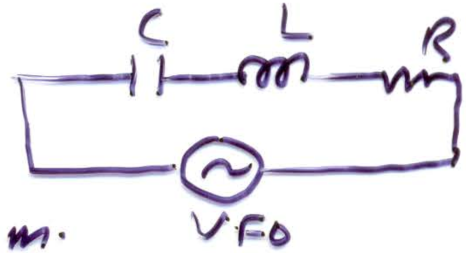




Resonance Curve
(Bell shape)

SERIES RESONANCE

when $X_C = X_L$ circuit is said to be at resonance. $Z = R$ is minimum.



$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$V_L = IX_L$, $V_C = IX_C$

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy dissipated / cycle}} = \frac{2\pi \frac{1}{2} L I^2}{\left(\frac{I}{\sqrt{2}}\right)^2 R / f}$$

$$= \frac{2\pi \cdot \frac{1}{2} L I^2 f}{\frac{I^2}{2} R} = \frac{2\pi f L}{R}$$

$$= \frac{X_L}{R}$$

$$= \frac{X_L}{R}$$

$V_L = IRQ$

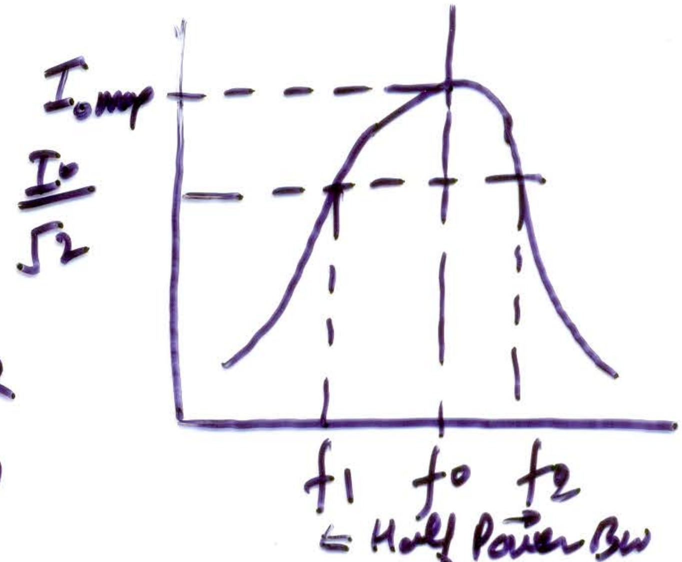
$V_L = VQ$ if $Q > 1$

cct acts as a magnifier of voltage.

$V_C = VQ$.

Relation between Half Power Bandwidth & Quality Factor for Series Resonant Circuit

At half power frequencies current is $\frac{1}{\sqrt{2}}$ of max.



ie $\frac{V}{R\sqrt{2}} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$

$X_L - X_C = \pm R$

$X_L - X_C = R$ at f_2

$X_C - X_L = R$ at f_1

$\therefore \omega_2 L - \frac{1}{\omega_2 C} = R$

multiply by $\frac{\omega_2}{L}$ $\quad \omega_2^2 - \frac{1}{LC} = \omega_2 \frac{R}{L} = \frac{\omega_2 \omega_0}{Q_0}$

$\frac{\omega_0 L}{R} = Q_0$

$\frac{R}{L} = \frac{\omega_0}{Q_0}$

ie $\omega_2^2 - \frac{1}{LC} = \omega_2 \frac{\omega_0}{Q_0}$ ——— (1)

$$\frac{1}{\omega_1 C} - \omega_1 L = R$$

multiply by $\frac{\omega_1}{L}$ $\quad \frac{1}{LC} - \omega_1^2 = R\frac{\omega_1}{L} = \omega_1 \frac{\omega_0}{Q_0}$ — (2)

ie $\quad \frac{1}{LC} - \omega_1^2 = \frac{\omega_1 \omega_0}{Q_0}$ — (2)

Adding (1) & (2)

$$\omega_2^2 - \omega_1^2 = \frac{\omega_0}{Q_0}(\omega_2 + \omega_1)$$

$$\omega_2 - \omega_1 = \frac{\omega_0}{Q_0}$$

$$f_2 - f_1 = \frac{f_0}{Q_0}$$

Half Power Band width $= \frac{f_0}{Q_0}$

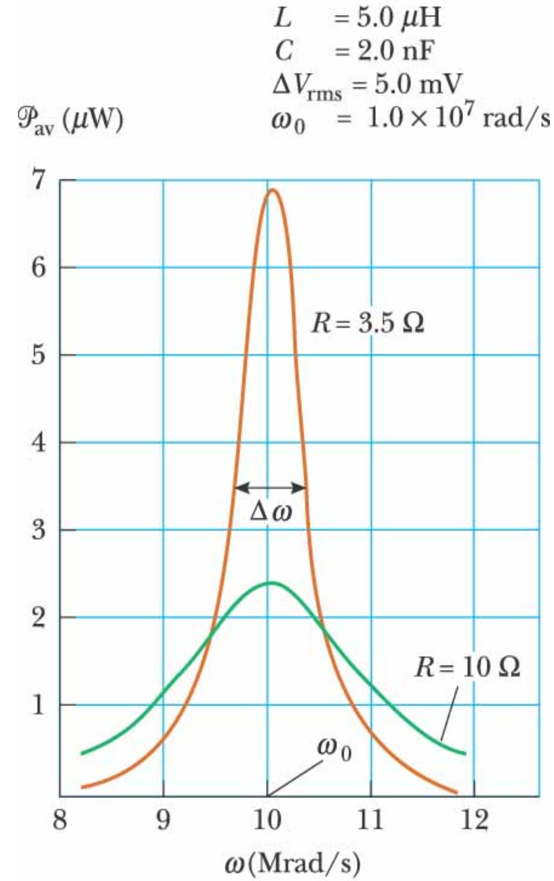
$$\boxed{BW = \frac{f_0}{Q_0}}$$

Power as a Function of Frequency

- Power can be expressed as a function of frequency in an *RLC* circuit

$$P_{av} = \frac{\left(\Delta V_{rms}\right)^2 R\omega^2}{R^2\omega^2 + L^2\left(\omega^2 - \omega_o^2\right)^2}$$

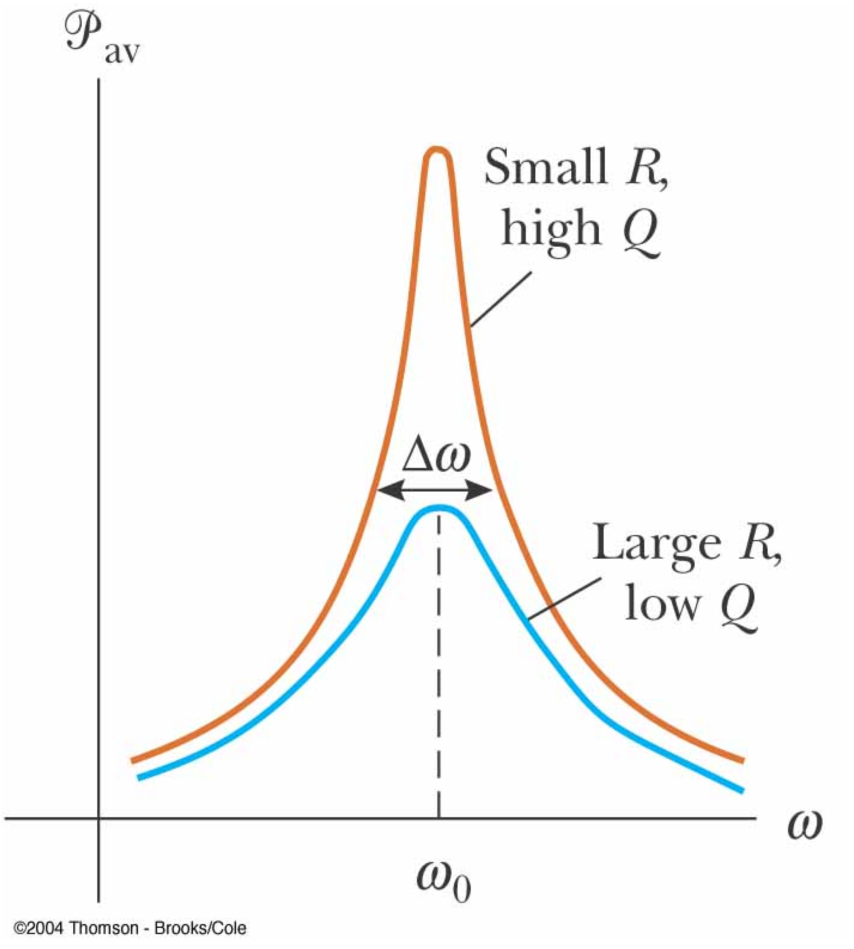
- This shows that **at resonance, the average power is a maximum**



(b)

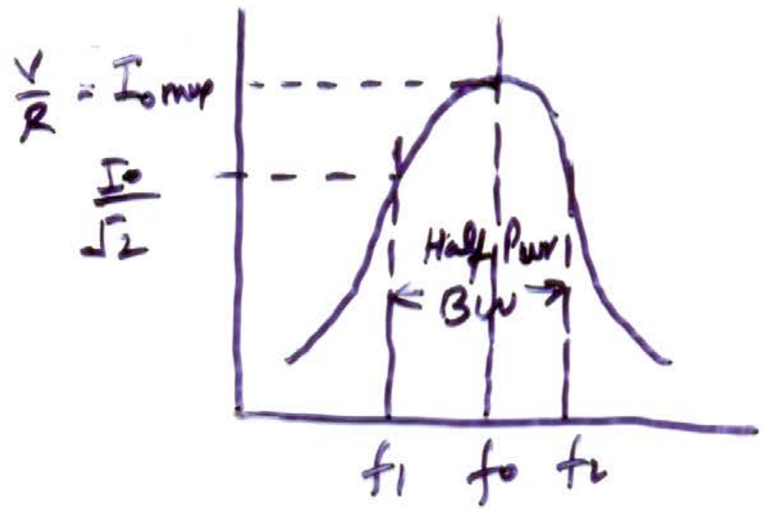
Quality Factor, cont.

- A high-*Q* circuit responds only to a narrow range of frequencies
  - Narrow peak
- A low-*Q* circuit can detect a much broader range of frequencies
- Typical *Q* values in electronics range from 10 to 100



$f_0 = \sqrt{f_1 f_2}$

At half power frequencies
Power is half or current
is $\frac{1}{\sqrt{2}}$ of max.



$$\frac{V}{\sqrt{2}\,R} = \frac{V}{\sqrt{R^2+(X_L-X_C)^2}}$$

or $2R^2 = R^2+(X_L-X_C)^2$

$X_L - X_C = \pm R$

below f_r, at f_1; $X_C - X_L = R$

above f_r at f_2 $X_L - X_C = R$

$X_C - X_L = R$ at f_1

$\frac{1}{\omega_1 C} - \omega_1 L = R$ at f_1

$\omega_2 L - \frac{1}{\omega_2 C} = R$ at f_2

$$\frac{1}{\omega_1 C} - \omega_1 L = \omega_2 L - \frac{1}{\omega_2 C}$$

$$\frac{1}{\omega_1 C} + \frac{1}{\omega_2 C} = \omega_2 L + \omega_1 L$$

$$= (\omega_1 + \omega_2) L$$

$$\frac{\omega_1 + \omega_2}{\omega_1 \omega_2 C}$$

$$\text{or } \frac{1}{\omega_1 \omega_2} = LC = \frac{1}{\omega_0^2}$$

$$\text{or } \omega_0^2 = \omega_1 \omega_2$$

$$\boxed{f_0^2 = f_1 f_2}$$

$$\boxed{f_0 = f_1 f_2}$$

resonance frequency of a Series RLC circuit is geometric mean of lower & upper half power frequencies.